

# A Node Splitting Interface Algorithm for Multi-rate Parallel Simulation of DC Grids

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**Abstract**—With wider applications of power electronic devices in modern power systems, simulation using traditional electro-mechanical and electromagnetic simulation tools suffer from low speed and imprecision. Multi-rate technologies can greatly improve simulation efficiency by avoiding simulating the entire system using a small time-step. However, the drawbacks of the current synchronization mechanisms is that they introduce numerical errors and numerical instabilities in multi-rate parallel simulations. An improved multi-rate parallel technology, node splitting interface (NSI), is proposed to reduce errors and enhance simulation stability. A new synchronization mechanism is used to avoid prediction and signal delays. Theoretical analyses are carried out to prove the convergence and absolute stability of the proposed NSI algorithm. This algorithm is particularly suitable for simultaneously investigating long term dynamics of DC grids and fast transients of power electronic converters.

**Index Terms**—DC grids, electromagnetic transient analysis, multi-rate interface, parallel algorithm, power system simulation.

## I. INTRODUCTION

THE key features of modern power systems include large-scale, complexity and high integration [1]. For example, a power system has more than 100 thousand buses, covering a scale of thousands of kilometers. Various power electronic systems, such as HVDCs and FACTS, are deeply integrated into the power system. These power electronics systems rely on precise control in microsecond scales [2], [3]. Therefore, a simulation of large-scale power systems with very small time-steps is required as part of the power system analysis tools [1].

Electro-mechanical simulation can easily simulate a large-scale power system but does not offer sufficient accuracy to mimic the response of power electronic devices due to its large time-step [4]. Electromagnetic transient programs enable small time-step simulations, but the small time-step simulation of a

large system will be inevitably time-consuming [5]. One of the solutions for modern power system simulations is to simulate a large-scale system with multiple time-steps. Different parts of a power system are modeled with different time-steps and linked to each other through an appropriate interface.

The electro-mechanical and electromagnetic hybrid simulation is applied in system-level power system studies [6]. The interface considering three sequential values can improve the simulation accuracy, however both a time-step delay and harmonics are missing and a solution is still unsolved [7].

Gear's multi-rate algorithm is based on implicit integration methods [8], [9]. State variables can be directly divided as fast variables and slow variables and computed with different time-steps [8]. Extrapolation strategies that affect the stability cannot be avoided [10]. The Waveform Relaxation method uses iterations to obtain the accurate results required, but iterations are not suitable for real-time simulations [12], [13].

The ideal transform interface (ITI) is a straightforward method [14], [15], in which external parts of a subnetwork are modeled as ideal sources. However, in this method, node signals (voltages and currents) at both sides of the interface are not guaranteed to be equal due to the ideal source model. In addition, node voltage signals are exchanged with a one time-step delay and are not synchronized. The accuracy of this method is greatly affected.

The transmission line interface (TLI) is an improved and widely applied method in AC systems [16], [17]. Transmission lines can be used to decouple a network. The natural delay characteristics of transmission lines can help to synchronize subnetworks without incurring any delay. A sampling scheme of the TLI is proposed to transfer data through filter banks [18]. This scheme can overcome the limitations for modeling linear, time-invariant subnetworks and increase the simulation time-step [18]. If no real transmission lines are available, inductors or capacitors are considered as equivalent transmission lines to decouple networks [19]. This equivalence could cause errors.

The TEC (Thevenin equivalent circuit) uses Thevenin equivalent sources to replace ideal electrical sources. This achieves more precise representation of both voltages and currents at the interface [20], [21]. However, this method is originated from single-rate applications. In multi-rate simulations, the TEC method is improved to induce the root-machine integration to suppress the numerical oscillation. The piecewise cubic spline function is substituted for the linear prediction [22]. However,

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predicted values from large time-step networks have to be used for small time-step networks [23], [24].

The SSN (state-space nodal) method is a parallel simulation method that has already been used in commercial simulation products [25]. However, the SSN is designed for single-rate parallel simulations. It requires a serial calculation step to solve the equations of all nodes for the interface [25]. This limits the application of the SSN to multi-rate parallel simulations.

So far multi-rate parallel simulations cannot achieve the same level of accuracy and stability as the traditional EMT with a single rate parallel simulation or without parallel simulation.

This paper proposes a novel multi-rate parallel algorithm, Node-Splitting Interface (NSI), which can use any node to decouple a network without the need of transmission lines. The advantage of this method is to avoid using prediction or signal delay. A backward mechanism is used to select proper history data from other subnetworks, and to synchronize the end time in each simulation step. Also, the serial procedure for solving the global nodal equation can be avoided.

Accuracy and stability of the proposed algorithm will be theoretically proved.

The NSI is tested using EMT in a large-scale simulation platform ADPSS [26], which has been used widely by academic and industrial organizations in China. Detailed comparisons with PSCAD/EMTDC for both simple and complex systems are carried out to verify the accuracy and efficiency of the proposed algorithm.

## II. FUNDAMENTAL THEORY

### A. Backward Synchronization

In Fig. 1, there are a voltage source,  $V_s$ , an inductor,  $L$ , a capacitor,  $C$ , and three resistors,  $R_1$ ,  $R_2$ ,  $R_{par}$ . In order to realize the multi-rate parallel simulation, the circuit is divided into two systems,  $s$  and  $f$ , with an interface of  $AA'$ .

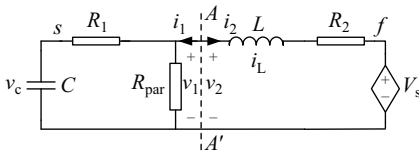


Fig. 1. Example of system.

System equations of the circuit are given in (1) to (3), in which equation (3) is an interface equation:

$$\begin{cases} C \frac{dv_c}{dt} = (v_1 - v_c) \frac{1}{R_1} \\ i_1 = \frac{v_1}{R_{par}} + \frac{v_1 - v_c}{R_1} \end{cases} \quad (1)$$

$$L \frac{di_2}{dt} = v_2 - V_s - R_2 i_2 \quad (2)$$

$$\begin{cases} v_2 = v_1 \\ i_1 = -i_2 \end{cases} \quad (3)$$

In a single time-step simulation, differential equations in (1) and (2) are discretized using the Backward Euler and the discretized equations are then:

$$\begin{cases} v_c(t + \Delta t) = \left(1 + \frac{\Delta t}{CR_1}\right)^{-1} v_c(t) \\ \quad + \left(1 + \frac{\Delta t}{CR_1}\right)^{-1} \left(\frac{\Delta t}{CR_1}\right) v_1(t + \Delta t) \\ i_1(t + \Delta t) = \frac{v_1(t + \Delta t)}{R_{par}} + \frac{v_1(t + \Delta t) - v_c(t + \Delta t)}{R_1} \\ i_2(t + \Delta t) = \left(1 + \frac{\Delta t}{LR_2}\right)^{-1} (i_2(t) \\ \quad + \frac{\Delta t}{L}(v_2(t + \Delta t) - V_s(t + \Delta t))) \end{cases} \quad (4)$$

The five algebraic equations in (3) and (4) with the five variables at  $t + \Delta t$  can be solved by assuming the history values at  $t$  are known. The solutions of the two subsystems are synchronized [27].

In the multi-rate parallel simulation, a large time-step,  $\Delta T$ , is used to discretize the differential equation of  $v_c$ , e.g. due to a large capacitor,  $C$ . And a small time-step,  $\Delta t$ , is used for the inductor equation:

$$\begin{cases} v_c(t + \Delta T) = \left(1 + \frac{\Delta T}{CR_1}\right)^{-1} v_c(t) \\ \quad + \left(1 + \frac{\Delta T}{CR_1}\right)^{-1} \left(\frac{\Delta T}{CR_1}\right) v_1(t + \Delta T) \\ i_1(t + \Delta T) = \frac{v_1(t + \Delta T)}{R_{par}} + \frac{v_1(t + \Delta T) - v_c(t + \Delta T)}{R_1} \\ i_2(t + \Delta t) = \left(1 + \frac{\Delta t}{LR_2}\right)^{-1} \\ \quad \left(i_2(t) + \frac{\Delta t}{L}(v_2(t + \Delta t) - V_s(t + \Delta t))\right) \end{cases} \quad (5)$$

In this case, the time of the variables in the  $s$  and  $f$  subsystems are different, the two subsystems are not synchronized. Therefore equation (3) is not fulfilled.

A straightforward method to achieve the synchronization is to introduce a delay of  $\Delta T$  for the interface variables,  $v_1$  and  $i_2$ , which means that  $v_1(t + \Delta T) = v_2(t)$  and  $i_2(t + \Delta T) = i_1(t)$  [24]. However, this method will cause large calculation errors and simulation instability.

The TEC method uses  $v_1(t + \Delta T) = v_2(t + \Delta t)$  to replace  $v_1 = v_2$  [27]. However, the replacement requires an approximation. Also, there exists a risk of increasing errors with the approximation.

This paper proposes a NSI algorithm, which selects proper history data from subsystems with different time-steps, in order to achieve synchronization.

The synchronization mechanism of this algorithm is listed as follows:

1) History data at  $t + \Delta t - \Delta T$  of  $v_c$ ,  $v_c(t + \Delta t - \Delta T)$ , is used to substitute  $v_c(t)$  in the first term of the right hand of the first equation of (5).

2)  $v_c(t + \Delta T)$  and  $v_1(t + \Delta T)$  in the first equation of (5) are replaced by  $v_c(t + \Delta t)$  and  $v_1(t + \Delta t)$ , respectively.

3)  $i_1(t + \Delta T)$ ,  $v_c(t + \Delta T)$  and  $v_1(t + \Delta T)$  are replaced by  $i_1(t + \Delta t)$ ,  $v_c(t + \Delta t)$  and  $v_1(t + \Delta t)$  in the second equation of (5).

4) Then, the interface variables in the slow subsystem become  $i_1(t + \Delta t)$  and  $v_1(t + \Delta t)$ , which are synchronized with  $i_2(t + \Delta t)$  and  $v_2(t + \Delta t)$  in the fast subsystem. Equation (3) is thus fulfilled.

5) According to (3), the synchronized solution of the entire network,  $v_c(t + \Delta t)$  and  $i_2(t + \Delta t)$ , can be achieved.

This mechanism uses backward synchronization. In summary, when the solution at  $t$  has been solved, the slow subsystem must select the history data at the point of  $t + \Delta t - \Delta T$  and transfer this data to the fast subsystem for further simulation.

Due to the large time constant, the discrete equation of  $v_c$  can be executed for every  $\Delta T$ , and its unknown value in every  $\Delta t$  is achieved through a linear interpolation, which can provide feasible accuracy [28].

In the first time-step, the initial history data at the point of 0 should be given as the condition to solve equations. There are two methods to access the initial history data. First, the solved system has to start from the zero condition and the initial history data of the state variables are set as zero. Secondly, the solved system has been running for a period and all variables in this period have become available. Except for these two methods, the history data should be predicted, which brings the approximations of the solution.

Furthermore, this method avoids the serial procedure in each time-step. This whole parallel procedure is suited for extending CPU scales to speed up the simulation and to achieve real-time simulation.

### B. General Modeling for NSI Algorithm

A power system is divided into two sub-networks, which connect to each other at  $n$  nodes, forming an interface,  $AA'$ , as shown in Fig. 2.

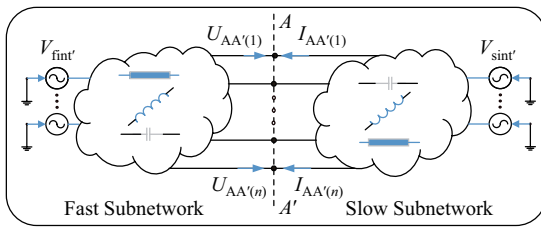


Fig. 2. Partition system by node direct decoupling.

One of the two sub-networks has fast dynamics with a small time constant, represented by subscript,  $f$ . Another subnetwork has slow dynamics with a large time constant, represented by subscript,  $s$ .

The state space equations of the two sub-networks are:

$$\begin{cases} \dot{X}_f = A_f X_f + [B_{fint} \ B_{fAA'}] [U_{fint} \ U_{fAA'}]^T \\ Y_{fAA'} = C_f X_f + [D_{fint} \ D_{fAA'}] [U_{fint} \ U_{fAA'}]^T \end{cases} \quad (6)$$

$$\begin{cases} \dot{X}_s = A_s X_s + [B_{sint} \ B_{sAA'}] [U_{sint} \ U_{sAA'}]^T \\ Y_{sAA'} = C_s X_s + [D_{sint} \ D_{sAA'}] [U_{sint} \ U_{sAA'}]^T \end{cases} \quad (7)$$

$U_{fint}$  and  $U_{sint}$  represent input variables inside each sub-network, which are the internal sources of each sub-network,  $V_{fint}$  and  $V_{sint}$ , in Fig. 2.  $U_{fAA'}$  and  $U_{sAA'}$  are input variables from the interface which represent the voltages,  $V_{AA'}(1)$  to  $V_{AA'}(n)$ , at the two sides respectively.  $Y_{fAA'}$  and  $Y_{sAA'}$  represent output variables, which are the currents cross the interface,  $I_{AA'}(1)$  to  $I_{AA'}(n)$  or  $-I_{AA'}(1)$  to  $-I_{AA'}(n)$ .  $A$ ,  $B$ ,  $C$ ,  $D$  are corresponding state-space matrix.

The relationships of the input variables and output variables are given in (8):

$$\begin{cases} U_{fAA'} = U_{sAA'} \\ Y_{fAA'} = -Y_{sAA'} \end{cases} \quad (8)$$

The issues of synchronization will be induced using the different mechanisms to deal with this relationship. Simplification or a delay of  $\Delta T$  is usually needed for solving the sub-networks, which would lead to large errors. The NSI algorithm can fulfil this relationship without any simplification.

The fast sub-network uses a simulation time-step  $h$ , and the slow sub-network uses a simulation time-step  $H = mh$ , where  $m$  is an integer. The backward synchronization mechanism is used to select proper history values, so that different equations from the state-space equations (6) and (7) can be achieved using the Backward Euler.

$$\begin{cases} X_f(mk) = (I - hA_f)^{-1} X_f(mk - 1) + (I - hA_f)^{-1} h \\ \quad [B_{fint} \ B_{fAA'}] [U_{fint}(mk) \ U_{fAA'}(mk)]^T \\ Y_{fAA'}(mk) = C_f X_f(mk) + [D_{fint} \ D_{fAA'}] \\ \quad [U_{fint}(mk) \ U_{fAA'}(mk)]^T \end{cases} \quad (9)$$

$$\begin{cases} X_s(mk) = (I - mhA_s)^{-1} X_s(mk - m) + (I - mhA_s)^{-1} \\ \quad mh [B_{sint} \ B_{sAA'}] [U_{sint}(mk) \ U_{sAA'}(mk)]^T \\ Y_{sAA'}(mk) = C_s X_s(mk) + [D_{sint} \ D_{sAA'}] \\ \quad [U_{sint}(mk) \ U_{sAA'}(mk)]^T \end{cases} \quad (10)$$

According to (8),  $U_{fAA'}(mk) = U_{sAA'}(mk)$  and  $Y_{fAA'}(mk) = Y_{sAA'}(mk)$ , thus these interface variables in (9) and (10) can be eliminated.

$$\begin{cases} X_f(mk) = A_{ff} X_f(mk - 1) + A_{fs} X_s(mk - m) \\ \quad + B_{fs} U_{sint}(mk) + B_{ff} U_{fint}(mk) \\ X_s(mk) = A_{sf} X_f(mk - 1) + A_{ss} X_s(mk - m) \\ \quad + B_{ss} U_{sint}(mk) + B_{sf} U_{fint}(mk) \end{cases} \quad (11)$$

where  $A_{ff}$ ,  $A_{fs}$ ,  $A_{ss}$ ,  $A_{sf}$ ,  $B_{ss}$ ,  $B_{sf}$ ,  $B_{ff}$  and  $B_{fs}$  are coefficient matrices, which are presented in Appendix B.

The state variables in the large time constant system change slowly. Therefore, the value in the  $mk + i$ th step between  $mk$  and  $mk + m$ th steps is obtained through interpolation. The linear interpolation is selected [20], as shown in (12), to support the sufficient accuracy:

$$X_s(mk + i) = \frac{m - i}{m} X_s(mk) + \frac{i}{m} X_s(mk + m) \quad (12)$$

As the sources,  $U_{\text{fint}}$  and  $U_{\text{sint}}$ , are known, the variables in the right side of the equation (11) are available in the  $k$ th calculation. The system has been divided into two decoupled synchronized subsystems represented by state transfer formulas with different recursive rates.

The features of the NSI are:

- 1) Different time-steps are used for state variables of different time constants to improve the simulation efficiency.
- 2) Interface relationships can be fulfilled in each step, to ensure the simulation accuracy.
- 3) Implicit integration without prediction can be easily formulated and implemented.

This multi-rate algorithm can easily be extended to the application of large-scale systems. A large-scale system should be divided into several layers according to their suitable time-steps. In each layer, there are many subsystems, which are synchronized using the parallel methods with the same time-step. Then, the information of the subsystems between different layers can be synchronized using the proposed multi-rate algorithm. Eventually, full system synchronization will be achieved. However, a large-scale system can normally be divided into three or four layers with different time-steps, such as 10 ms, 50  $\mu$ s and 1  $\mu$ s or 1 minute, 10 ms, 50  $\mu$ s and 1  $\mu$ s.

### III. ACCURACY ANALYSIS

#### A. Accuracy Analysis of NSI

The accuracy and stability of multi-rate parallel algorithms are expected to be similar to that of a traditional EMT. However, the major drawback of existing multi-rate parallel algorithms is lack of theoretical analyses on this issue.

In this section, the accuracy of NSI is proved. The conditions of the proof will be clearly given. Meanwhile, the expression of errors is given to study factors which influence the errors.

As long as the consistency and stability of this integration are proved, the convergence of the integration can be achieved [28]. In this section, the consistency of errors will be proved first.

**Theorem 1.** *Suppose that the original system is asymptotic stability, and the divided fast and slow subsystems are also asymptotic stability. All time-steps of the subsystems are fixed through the simulation. Then, the error of the solution of NSI is globally consistent.*

The errors,  $\xi$ , can be expressed as (13), whose derivation is given in the following section:

$$\xi(Hk) < \|R(m)\| K H e^{LHk} - \|R(m)\| K H \quad (13)$$

In (13),  $\|R(m)\|$  is constant for a given  $m$ .  $K$  and  $L$  are constants, which can be calculated with the system parameters, as shown in “proof of consistency.”  $k$  represents the  $k$ th step in the simulation.

This means that the simulation error at any step is only determined by the  $L$  and  $K$ , which are time-invariant, and related to the norms of the coefficient matrix of the original system. If the simulation step  $H$  is small enough, the errors can be limited within any allowed margin at a certain time.

#### B. Proof of Consistency

In order to find a unified expression of maximum errors of the NSI algorithm in each step, a recursive expression is derived first. The detailed proof of consistency is demonstrated in Appendix C.

The derived recursive error expression  $e_{\text{xf}}$  for the fast subnetwork and  $e_{\text{xs}}$  for the slow subnetwork, can be obtained as equation (14) and (15):

$$e_{\text{xf}}(mk + i) = A_{\text{ff}}^i e_{\text{xf}}(mk) + \sum_{j=0}^{i-1} A_{\text{ff}}^{i-1-j} A_{\text{fs}} e_{\text{xs}}(mk - m + j) \quad (14)$$

$$e_{\text{xs}}(mk + m) \leq A_{\text{sf}} A_{\text{ff}}^{m-1} e_{\text{xf}}(mk) + A_{\text{sf}} m A_{\text{fs}} \|R(m)\| O(mh) + \left( A_{\text{sf}} K p \sum_{i=0}^{m-1} A_{\text{ff}}^{m-1-i} A_{\text{fs}} \right) e_{\text{xs}}(mk) + A_{\text{ss}} e_{\text{xs}}(mk) \quad (15)$$

where  $\|R(m)\|O(mh)$  represents the local truncation error, and  $O(mh)$  means the infinity of a large time-step,  $mh$ .  $Kp$  is generally determined by the interpolation method and is less than 2 [28].

The unified form of the maximum of  $e_{\text{xf}}$  and  $e_{\text{xs}}$ ,  $\xi$ , is given in equation (16):

$$\xi(mk) < \sum_{j=0}^{k-1} Lmh(\xi((k-1-j)m) + \|R(m)\| Kmh) \quad (16)$$

where  $L$  and  $K$  are the coefficients. The detailed information is demonstrated in Appendix C.

And then, an eventual expression can be achieved through removing the recursive terms.

The representation of the error  $\xi$  will change from discrete points,  $\xi(mk)$ , to the time series,  $\xi(mkh)$ . Obviously, this summation series formula can be changed to the exponent expression that,

$$\xi(Hk) < \|R(m)\| K H e^{LHk} - \|R(m)\| K H \quad (17)$$

Taking  $\tau = kH$ , the error  $\xi(\tau) \rightarrow 0$  from this inequality, when  $h \rightarrow 0$ . Therefore, the global error is consistent.

### IV. STABILITY ANALYSIS

#### A. Absolute Stability of NSI

Generally, major multi-rate parallel algorithms cannot guarantee the absolute stability [8]. However, the proposed NSI can be proved to be stable. Conditions of this proof are also clearly given. The stability can support the convergence of the NSI algorithms.

**Theorem 2.** *Suppose that the original single time dimension system is asymptotic stability, and the divided fast subsystems are also asymptotic stability, then the new system with the proposed multi-rate simulation algorithm is stable.*

It means errors caused by any disturbance will not be amplified. The proof of this theorem is given in the next section.

Based on the absolute stability and consistency, the convergence of error can be eventually achieved. This means that, as long as  $h$  is small enough, the errors will be limited within any allowed margin at any time.

### B. Proof of Stability

In order to prove the stability of NSI, one step of the multi-rate integration is divided into several small-rate steps and a two-rate step. The two-rate step is a step of integration with a small-rate for some variables and a large-rate for other variables.

If every small-rate step is stable and the two-rate step is stable, the entire multi-rate integration must be stable.

Therefore, the first stage is to divide the integration.

The state transfer equation of the whole system is that:

$$\begin{bmatrix} X_f(mk+m) \\ X_s(mk+m) \\ X_s(mk) \end{bmatrix} = \Phi_s \prod_{i=1}^{m-1} \Phi_f(i) \begin{bmatrix} X_f(mk) \\ X_s(mk) \\ X_s(mk-m) \end{bmatrix} \quad (18)$$

where  $\Phi_f(i)$  represents the transfer matrix at the  $i$ th small step within one large step, as shown below:

$$\Phi_f(i) = \begin{bmatrix} A_{ff} & \frac{i}{m} A_{fs} & \frac{m-i}{m} A_{fs} \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \quad (19)$$

$\Phi_s$  represents the transfer matrix of the two-rate integration:

$$\Phi_s = \begin{bmatrix} A_{ff} & A_{fs} & 0 \\ A_{sf} & A_{ss} & 0 \\ 0 & I & 0 \end{bmatrix} \quad (20)$$

The criteria [31] to determine whether the system is stable is shown as:

$$\rho \left( \Phi_s \prod_{i=1}^{m-1} \Phi_f(i) \right) \leq \left\| \Phi_s \prod_{i=1}^{m-1} \Phi_f(i) \right\|_p \leq 1 \quad (21)$$

It is obvious that the norms of  $A_{ff}$  are less than 1. Thus, the norms of  $\Phi_f(i)$  are less than 1.

The proof that the norm of  $\Phi_s$  is less than 1 is demonstrated in Appendix D.

$$\|\Phi_s\| < 1 \quad (22)$$

Therefore, it is given that

$$\Phi_s \prod_{i=1}^{m-1} \Phi_f(i) < 1 \quad (23)$$

This proves the NSI algorithm is stable.

## V. CASE STUDIES

Comparisons of accuracy and stability between NSI results and accurate results are performed through a simple case and an integrated AC/MTDC system. Accurate results are achieved through the simulation with a very small single time-step. The simple case provides comparisons of various multi-rate parallel algorithms, TLI, ITI and NSI, and the complicated case only verifies the accuracy of the NSI algorithm.

In the simple case, the accuracy is evaluated with different time-steps. Multi-rate results and accurate results are all

achieved using MATLAB/Simpower. In the integrated AC/MTDC system, a large disturbance is implemented to evaluate errors between the accurate results and NSI results. PSCAD 4.5 is utilized to obtain accurate results, and NSI results come from ADPSS.

### A. Accuracy Evaluation of NSI

Results of the NSI are compared with accurate results on the circuit in Fig. 1. Results of ITI and TLI are also provided in this section as a comparison for the performance of different multi-rate parallel algorithms. In the base cases, two rates of 1  $\mu$ s and 50  $\mu$ s were used. The fundamental frequency of the source is 50 Hz.

The TLI algorithm requires an inductor,  $L_{TLI}$ , to decouple the circuit in Fig. 3. So the original inductor,  $L$ , is divided into two inductors,  $L_{TLI}$  and  $L'$ , and  $L = L_{TLI} + L'$ . The parameters of the circuit are given in Table I.

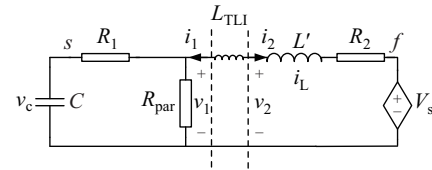


Fig. 3. Transmission line interface.

TABLE I  
PARAMETERS OF THE CIRCUIT

| $R_1/\Omega$ | $C/F$                    | $R_{par}/\Omega$ | $R_2/\Omega$ |
|--------------|--------------------------|------------------|--------------|
| 110          | 0.001                    | 108              | 110          |
| $L/H$        | $V_s/V$                  | $L_{TLI}/H$      |              |
| 0.101        | $\sin(50 \times 2\pi t)$ | 0.05             | —            |

The results of  $i_2$  in the base cases are shown in Fig. 4. Errors represent the difference between the results of a multi-rate parallel algorithm and the accurate results. The TLI has the largest rate of error, up to 8%, and the ITI has a rate of error of only 2.5%. The rate of error of NSI is less than 0.005%.

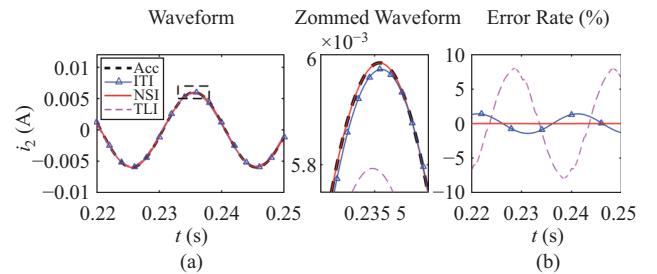


Fig. 4. Results of the rates of 1  $\mu$ s and 50  $\mu$ s. (a) Currents in different simulation methods. (b) Rate of error (Multi-rate-ACC/RMS of ACC).

In order to evaluate the influences of increasingly large time-steps, a large time-step of 100  $\mu$ s is used instead of 50  $\mu$ s. The results are shown in Fig. 5. The ITI is unstable, so its error is not shown in Fig. 5(b). The TLI remains stable. However, its rate of error increases to almost 13%, which is not acceptable. Again, the NSI achieves very high accuracy, but this error increases to 0.01%, which is 2 times that of the previous case.

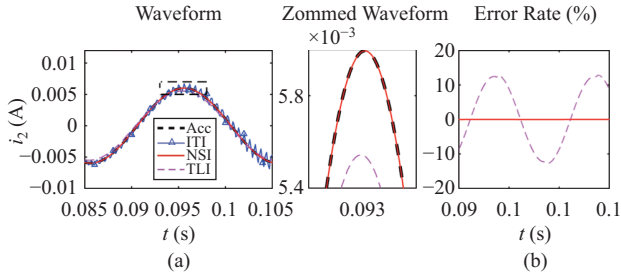


Fig. 5. Increasing large time-steps (the rates of  $1 \mu\text{s}$  and  $100 \mu\text{s}$ ). (a) Currents in different simulation methods. (b) Rate of error (Multi-rate-ACC/RMS of ACC).

In order to evaluate the impact of reducing large time-steps, the large time-step is changed from  $50 \mu\text{s}$  to  $10 \mu\text{s}$ . The results are shown in Fig. 6. The ITI is stable, and its error reduces to 0.7%. The TLI has a peak error of 7%, the NSI exhibits the smallest error, reduced to 0.001%.

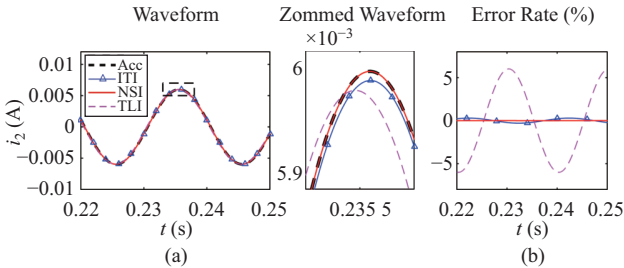


Fig. 6. Reducing large time-steps (the rates of  $1 \mu\text{s}$  and  $10 \mu\text{s}$ ). (a) Currents in different simulation methods. (b) Rate of error (Multi-rate-ACC/RMS of ACC).

In conclusion, there are limitations with the existing multi-rate parallel algorithms, ITI and TLI. The stability of ITI is significantly affected by the time-steps. The TLI is much more stable than the ITI, however, its accuracy is unsatisfactory. In contrast, the proposed NSI maintains a high accuracy and stability in all tests. The large time-steps and errors show the positive correlation.

Although the stability of the NSI has not been proved for unstable systems, the simulation using NSI for an unstable system is acceptable, because the NSI achieves smaller errors than the ITI and TLI which have already been widely used for practical applications. Furthermore, the realistic system has unstable frequencies from 0 Hz to 1000 Hz. To achieve an accurate result, simulation rates should be much faster than those unstable frequencies and the typical rate is 20 kHz.

### B. Application of NSI in an Integrated AC/MTDC System

An integrated AC/MTDC system, as shown in Fig. 8, is used to evaluate the performance of NSI. The system consists of two IEEE 9-Bus systems and a five terminal HVDC network. This system is divided into two subsystems along the dashed line box. The two IEEE 9-Bus systems outside the box are chosen as a slow subsystem with the time-step of  $50 \mu\text{s}$ , while the MTDC system inside the box is a fast subsystem with the time-step of  $2 \mu\text{s}$ .

The MTDC grid system refers to the Zhoushan 5 terminal

HVDC projects [32]. Two-level converters are used in each VSC, instead of MMC, as converter topology are not the research topic of this paper and do not affect the multi-rate parallel algorithm analysis. Also, line models with propagation delay are used for the DC submarine cables. All the converters use the current loop control. Detailed parameters are listed in Appendix A.

In IEEE 9-Bus systems, AC transmission lines are modeled as PI sections. Generators are modeled as a  $dq0$  model, represented by Park equations with a AVR and a governor. Detailed parameters are shown in [33].

Results of PSCAD and the NSI are compared under AC and DC faults. A single-phase fault is set at ABUS 9 at 2 s and it lasts 200 ms.

The fault resistance is  $8 \Omega$ . The performance of CS1 is plotted to show the simulation accuracy. The AC voltage, at ABUS 9, and the AC current, flowing into CS1 from ABUS9, are plotted in Fig. 7(a) and (b). The right plot is the zoomed waveform of the left plot. The DC positive voltage of CS1 and the current between the converter and capacitor are plotted in Fig. 7(c) and (d). The results show that the two curves almost coincide with each other. It verifies that NSI achieves a high accuracy.

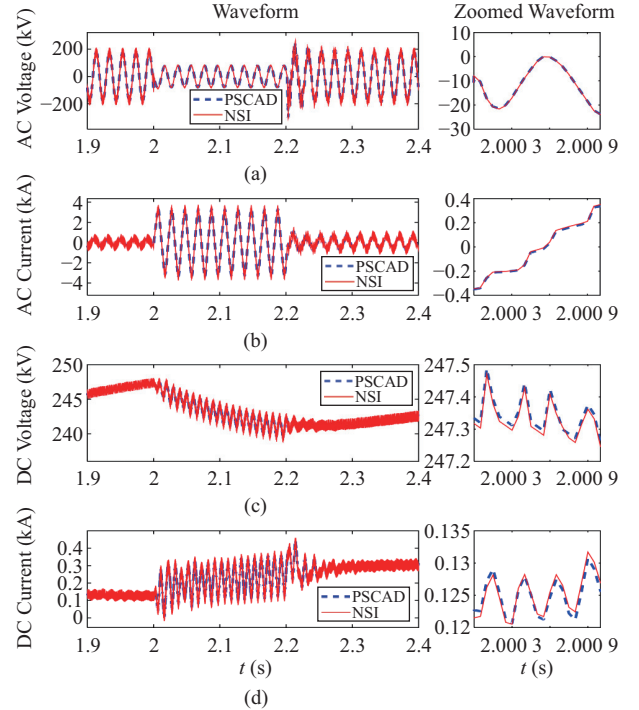


Fig. 7. Results of AC single-phase fault. (a) Voltage of ABUS 9. (b) AC current injected to CS1. (c) DC positive voltage of CS1. (d) Current between the converter and the capacitor in CS1.

DC short circuit occurs at CS4. The results in Fig. 9 show that the NSI can match PSCAD results well.

### C. Speeding up Simulation

This speeding up test is performed on a PC, in which the configuration is E5 2600, 8 GB DDR3 and 500 GB SAS.

Speeding up the simulation using NSI is illustrated in Fig. 10. The computation time of the simulation is compared

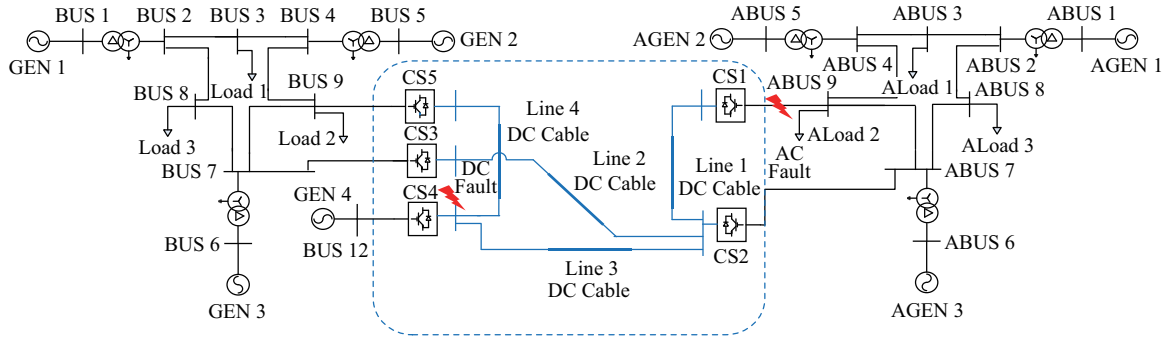


Fig. 8. Topology of the Integrated AC/MTDC system.

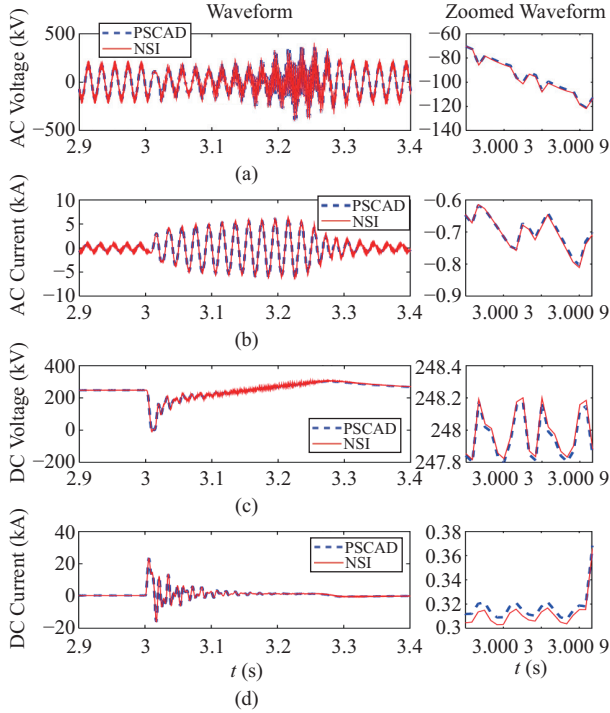


Fig. 9. Results of DC fault. (a) Voltage of ABUS 9. (b) AC current injected to CS1. (c) DC positive voltage of CS1. (d) Current between the converter and the capacitor in CS1.

under a different number of subsystems. The digit after  $S$  and  $F$  represents the number of slow subsystems and fast subsystems. For instance, S1:F1 means one slow subsystem and one fast subsystem, S1:F3 means one slow subsystem and three fast subsystems.

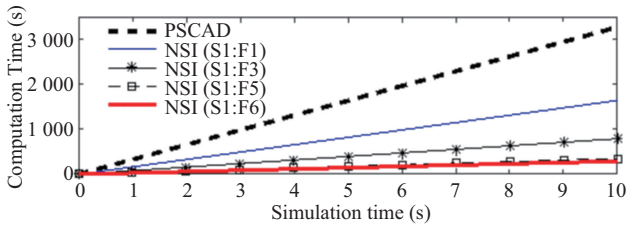


Fig. 10. Computation time of simulation under different quantities of subsystems.

The traditional simulation of the 10 s period takes about 3300 s and the proposed NSI takes 290 s, a speed up of

1,000%. Thus, the computational time for the simulation using NSI is much shorter than the traditional EMT simulation (PSCAD). Also, the computation time reduces along with the increase in the number of subsystems with small time-steps. When the subsystems are more than 6, the rate of speeding up will not be significantly increased.

## VI. CONCLUSION

This paper analyzes in-depth multi-rate parallel interfaces for power system simulation and proposes the Node Splitting Interface algorithm.

The state space model of the Node Splitting Interface algorithm has been established to reduce simulation errors by selecting suitable history values.

To evaluate the feasibility of this method, the error analysis is carried out to prove the accuracy of the algorithm.

The stability of this method is studied. As long as the original system is stable, the multi-rate parallel simulation is stable as well.

By summarizing the theoretical analyses, the accuracy theorem and the stability theorem have been proposed and proved.

In order to compare the proposed method and existing methods, simulations on simple circuits have been performed using different large time-steps. Another case study with a 5 terminal MTDC shows that the Node Splitting Interface algorithm can achieve high accuracy in the DC grid simulation, as well as a high efficiency.

The NSI is particularly suitable for multi-terminal HVDC grids to investigate both fast transient responses in the range of micro-seconds of multiple converters and long term dynamics in ranges of tens of seconds of whole systems. This method will benefit the study of large systems with multiple FACTS and HVDC links.

## APPENDIX

### A. Parameters of Zhoushan MTDC

Some parameters of the VSC-MTDC in Fig. 8 on the large case study are listed below, the detailed information refers to [34].

**Transmission Line:** Line 1 = 46 km, Line 2 = 17 km, Line 3 = 39 km, Line 4 = 32.3 km,  $R = 0.0192$  ohm/km,  $L = 0.24$  mH/km,  $C = 0.2961$  uF/km.

**Converter:**

Zhoushan (CS1) (VQ mode),  $V = \pm 200$  kV,  $Q = 60$  MVar  
Capacitor = 432 uF;

Daishan (CS2) (PQ mode),  $P = -200$  MW,  $Q = 60$  MVar;  
Capacitor = 432 uF;

Qushan (CS3) (PQ mode),  $P = 80$  MW,  $Q = 20$  MVar;  
Capacitor = 108 uF;

Yangshan (CS4) (PQ mode),  $P = 80$  MW,  $Q = 20$  MVar;  
Capacitor = 108 uF;

Shensi (CS5),  $P = 80$  MW,  $Q = 20$  MVar; Capacitor =  
108 uF;

**B. Expression of the Coefficiency Matrices in Eq. (11)**

$$\begin{aligned} A_{ff} &= A_{f-dis} - B_{fAA'-dis} \Delta C_f A_{f-dis} \\ A_{fs} &= -B_{fAA'-dis} \Delta C_s A_{s-dis} \\ A_{ss} &= -B_{sAA'-dis} \Delta C_s A_{s-dis} \\ A_{sf} &= -B_{sAA'-dis} \Delta C_f A_{f-dis} \\ B_{ff} &= B_{fint-dis} - B_{fAA'-dis} \Delta (C_f B_{fint-dis} + D_{fint}) \\ B_{fs} &= -B_{fAA'-dis} \Delta (D_{sint} + C_s B_{sint-dis}) \\ B_{ss} &= B_{sint-dis} - B_{sAA'-dis} \Delta (C_s B_{sint-dis} + D_{sint}) \\ B_{sf} &= -B_{sAA'-dis} \Delta (D_{fint} + C_f B_{fint-dis}) \end{aligned}$$

And  $\Delta = (C_s B_{sAA'-dis} + D_{fAA'} + D_{sAA'} + C_f B_{fAA'-dis})^{-1}$   
where:

$$\begin{cases} A_{s-dis} &= (I - mhA_s)^{-1} \\ B_{sint-dis} &= (I - mhA_s)^{-1} mhB_{sint} \\ A_{f-dis} &= (I - hA_f)^{-1} \\ B_{sAA'-dis} &= (I - mhA_s)^{-1} mhB_{sAA'} \\ B_{fAA'-dis} &= (I - hA_f)^{-1} hB_{fAA'} \\ B_{fint-dis} &= (I - hA_f)^{-1} hB_{fint} \end{cases} \quad (A1)$$

**C. Proof of Consistency**

Rewriting (11) obtains,

$$\begin{cases} X_f(mk+i) = A_{ff} X_f(mk+i-1) \\ \quad + A_{fs} \left( \frac{m-i}{m} X_s(mk-m) + \frac{i}{m} X_s(mk) \right) \\ \quad + B_{fs} U_{sint}(mk+i) + B_{ff} U_{fint}(mk+i) \\ X_s(mk+m) = A_{sf} X_f(mk+m-1) + A_{ss} X_s(mk) \\ \quad + B_{ss} U_{sint}(mk+m) + B_{sf} U_{fint}(mk+m) \end{cases} \quad (A2)$$

Convert (A2) to the error equation (A3):

$$\begin{cases} e_{xs}(mk+m) = A_{sf} e_{xf}(mk+m-1) + A_{ss} e_{xs}(mk) \\ e_{xf}(mk+i) = A_{ff} e_{xf}(mk+i-1) + A_{fs} e_{xs}(mk+i-m) \end{cases} \quad (A3)$$

The error of the fast system,  $e_{xf}(mk+i)$ , is achieved based on the values in the  $mk$ th step. The derived error expression can be obtained as equation (A4):

$$e_{xf}(mk+i) = A_{ff}^i e_{xf}(mk) + \sum_{j=0}^{i-1} A_{ff}^{i-1-j} A_{fs} e_{xs}(mk-m+j) \quad (A4)$$

In the formula above,  $e_{xs}(mk-m+j)$ , consists of two parts: the local truncation error from a single interpolation and the accumulative error from itself. In fact, the local truncation error of each  $m$  step is limited [25]. Also, it is assumed that the maximum accumulative error in the  $mk$ th step is  $e_{xs}(mk)$ , then, the accumulative error in the steps between the  $mk-m$ th and the  $mk$ th is less than  $Kp$  times the maximum error.  $Kp$  is generally determined by the interpolation method and is less than 2 [25]. Therefore, the error expression can be derived as follows:

$$\begin{aligned} e_{xs}(mk-m+i) &= e(mk-m+i) + e_{accu}(mk-m+i) \\ &< \|R(m)\| O(mh) + Kp e_{xs}(mk) \end{aligned} \quad (A5)$$

The local truncation error  $\|R(m)\| O(mh)$  is a function with a finite value of  $m$ , and  $h$  is a small simulation time-step, and  $O(mh)$  means the infinity of a large time-step,  $mh$ .

Combined with the equation (A4) and considering the synchronization step,  $mk$ th, the transfer error from the fast system is obtained as below:

$$\begin{aligned} e_{xf}(mk+m) &\leq A_{ff}^m e_{xf}(mk) + mA_{fs} \|R(m)\| O(mh) \\ &\quad + Kp \sum_{i=0}^{m-1} A_{ff}^{m-1-i} A_{fs} e_{xs}(mk) \end{aligned} \quad (A6)$$

Substituting (A6) to (A3), the global error for the slow system can be expressed as:

$$\begin{aligned} e_{xs}(mk+m) &\leq A_{sf} A_{ff}^{m-1} e_{xf}(mk) \\ &\quad + A_{sf} mA_{fs} \|R(m)\| O(mh) \\ &\quad + (A_{sf} Kp \sum_{i=0}^{m-1} A_{ff}^{m-1-i} A_{fs}) e_{xs}(mk) \\ &\quad + A_{ss} e_{xs}(mk) \end{aligned} \quad (A7)$$

Due to the absolute stability of every sub-network, the norms of  $A_{ff}$ ,  $\|A_{ff}\|$ , are less than 1. So it follows from (A6) that:

$$\begin{aligned} e_{xf}(mk+m) &\leq A_{ff}^m e_{xf}(mk) + m \|A_{fs}\| \|R(m)\| O(mh) \\ &\quad + m \cdot Kp \|A_{ff}\| \|A_{fs}\| e_{xs}(mk) \end{aligned} \quad (A8)$$

Due to the absolute stability of every sub-network, the norms of  $A_{ff}$ ,  $\|A_{ff}\|$ , are less than 1. Using the recursive formula (30) and (31), the final global error for the  $mk$ th step can be obtained from the initial values:

$$\begin{aligned} e_{xf}(mk) &\leq A_{ff}^m e_{xf}(0) + \sum_{j=0}^{k-1} A_{ff}^{(j)m} (m \|A_{fs}\| \|R(m)\| O(mh) \\ &\quad + m \cdot Kp \|A_{ff}\| \|A_{fs}\| e_{xs}((k-1-j)m)) \end{aligned} \quad (A9)$$

$$\begin{aligned} e_{xs}(mk) &\leq \left( \|A_{sf} Kp \sum_{i=0}^{m-1} A_{ff}^{m-1-i} A_{fs} + A_{ss} \| \right)^k e_{xs}(0) \\ &\quad + \sum_{j=0}^{k-1} A_{ss}^j (A_{sf} A_{ff}^{m-1} e_{xf}((k-1-j)m) \\ &\quad + A_{sf} mA_{fs} \|R(m)\| O(mh)) \end{aligned} \quad (A10)$$

It is assumed that the error is zero at the initial step, then:

$$\|A_{ff}\| < 1 \quad \|A_{ss}\| < 1 \quad \left\| A_{sf} K p \sum_{i=0}^{m-1} A_{ff}^{m-1-i} A_{fs} \right\| < 1$$

$$e_{xf}(0) = e_{xf}(0) = 0 \quad (A11)$$

Let  $\xi(km)$  be the maximum of global errors in the  $km$  th step, as shown below:

$$\xi((k-1-j)m) = \max\{e_{xs}((k-1-j)m), e_{xf}((k-1-j)m)\} \quad (A12)$$

From the definition of  $A_{sf}$  and  $A_{fs}$  in (A1), the norm of  $A_{sf}$  and  $A_{fs}$  has the proportion of  $mh$  and  $h$ , so (A13) and (A14) can be derived from (A9) and (A10):

$$e_{xs}(mk) < \sum_{j=0}^{k-1} Lmh(\xi((k-1-j)m) + \|R(m)\| K_s mh) \quad (A13)$$

$$e_{xf}(mk) < \sum_{j=0}^{k-1} Lmh(\xi((k-1-j)m) + \|R(m)\| K_f mh) \quad (A14)$$

The unified form of errors is given in equation (A15):

$$\xi(mk) < \sum_{j=0}^{k-1} Lmh(\xi((k-1-j)m) + \|R(m)\| K mh) \quad (A15)$$

where:

$$L = \max\{\|A_{ss}\| \|A_{sf}\| \|A_{ff}\|^{m-1}, \|A_{ff}\|^m K p \|A_{ff}\| m \|A_{fs}\|\} / mh \quad (A16)$$

$$K_f = \|A_{ff}\|^m m \|A_{fs}\| O(mh) / (L(mh)^2) \quad (A17)$$

$$K_s = \|A_{ss}\| \|A_{sf}\| m \|A_{fs}\| O(mh) / (L(mh)^2) \quad (A18)$$

and  $K$  is the larger one of  $K_s$  and  $K_f$ .

#### D. Proof of (22)

The original system (6)–(8) can be rewritten as (A19) by eliminating the variables of the interface inputs,  $U_{sAA'}$  and  $Y_{sAA'}$ .

$$\begin{bmatrix} \dot{X}_f \\ \dot{X}_s \end{bmatrix} = \begin{bmatrix} A_f - B_{fAA'}(D_f + D_s)^{-1} C_f - B_{fAA'}(D_f + D_s)^{-1} C_s \\ -B_{sAA'}(D_f + D_s)^{-1} C_f - B_{sAA'}(D_f + D_s)^{-1} C_s \end{bmatrix} \begin{bmatrix} X_f \\ X_s \end{bmatrix} \quad (A19)$$

The original system is absolute stable, thus the state matrix of (A20), denoted as  $A$ , is stable, whose eigenvalues have the negative real parts. By introducing a new matrix  $A_m$ ,

$$A_m = \begin{bmatrix} I & 0 \\ 0 & mI \end{bmatrix} A = \begin{bmatrix} A_f - B_{fAA'}(D_f + D_s)^{-1} C_f & -B_{fAA'}(D_f + D_s)^{-1} C_s \\ -mB_{sAA'}(D_f + D_s)^{-1} C_f & mA_s - mB_{sAA'}(D_f + D_s)^{-1} C_s \end{bmatrix} \quad (A20)$$

whose eigenvalues also have the negative real parts as well.

Using  $mB_{sAA'}$  and  $mA_s$  to replace the coefficient matrix of  $B_{sAA'}$  and  $A_s$  in the original system, the state matrix of the new system, as shown in (A21), is just  $A_m$ . Therefore, the new system is also absolute stable.

$$\begin{cases} \dot{X}_f = A_f X_f + B_{fAA'} U_{fAA'} & \dot{X}_s = mA_s X_s + mB_{sAA'} U_{sAA'} \\ Y_{fAA'} = C_f X_f + D_{fAA'} U_{fAA'} & Y_{sAA'} = C_s X_s + D_{sAA'} U_{sAA'} \\ U_{fAA'} = U_{sAA'} & Y_{fAA'} = -Y_{sAA'} \end{cases} \quad (A21)$$

Discretizing this system by a small time-step  $h$ , the transfer equations are obtained:

$$\begin{cases} X_f(k+1) = (A_{f-dis} - B_{fAA'-dis} \Delta C_f A_{f-dis}) X_f(k) \\ \quad - B_{fAA'-dis} \Delta C_s A_{s-dis} X_s(k) \\ X_s(k+1) = -B_{sAA'-dis} \Delta C_f A_{f-dis} X_f(k) \\ \quad + (A_{s-dis} - B_{sAA'-dis} \Delta C_s A_{s-dis}) X_s(k) \end{cases} \quad (A22)$$

And  $\Delta = (C_s B_{sAA'-dis} + D_{fAA'} + D_{sAA'} + C_f B_{fAA'-dis})^{-1}$   
The state transfer matrix of this equations is:

$$\Phi_{s11} = \begin{bmatrix} A_{f-dis} - B_{fAA'-dis} \Delta C_f A_{f-dis} & -B_{fAA'-dis} \Delta C_s A_{s-dis} \\ -B_{sAA'-dis} \Delta C_f A_{f-dis} & A_{s-dis} - B_{sAA'-dis} \Delta C_s A_{s-dis} \end{bmatrix} \quad (A23)$$

The norms of this matrix,  $\Phi_{s11}$ , are less than 1 due to the absolute stability of this new system.

$$\|\Phi_{s11}\| < 1 \quad (A24)$$

In fact, the coefficient of  $\Phi_{s11}$  is the same as a sub-matrix of  $\Phi_s$ , given in (20).

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} = \begin{bmatrix} A_{f-dis} - B_{fAA'-dis} \Delta C_f A_{f-dis} & -B_{fAA'-dis} \Delta C_s A_{s-dis} \\ -B_{sAA'-dis} \Delta C_f A_{f-dis} & A_{s-dis} - B_{sAA'-dis} \Delta C_s A_{s-dis} \end{bmatrix} \quad (A24)$$

The norms of  $\Phi_{s11}$  determine the norms of  $\Phi_s$  [28], meaning that if (A24) is stable, the (20) is stable as well.

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